647510 **BSc6MatC602x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination

March/April-2024 (NEW COURSE)

Mathematical Analysis - 2 and Abstract Algebra-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any two out of three. **(10)**

(1) Define compact set and using definition check whether (0,1) is compact or not.

(2) Prove that Non-empty subset E of discrete metric space is singleton iff E is connected.

(3) State and prove Heine-Borel theorem.

Que-1(B) Answer any one out of two. **(04)**

(1) Define Separated sets. A and B be two disjoint closed subsets of metric space X then show that A and B are separated sets.

(2) (i) Check whether set A={1,16,49,……..} is countable or not.

(ii) Prove that every compact metric space X is Totally bounded set.

Que-2(A) Answer any two out of three. **(10)**

(1) If f(t) is continuous for all $t \geq 0$ and f'(t) is piecewise Continuous then prove that L{f'(t)} Exists, provided $\lim_{t\to\infty}[e^{-st} f(t)]=0$ and

L{f'(t)}=sL{f(t)}-f(0)=s $\overline{f}$ (s)-f(0). Also find L[x^2sinhx].

(2) Prove: If $L\{f(t)\} = \bar{f}(s)$ then $L\{e^{at}f(bt)\} = \frac{1}{b}\,\bar{f}\left(\frac{s-a}{b}\right), b > 0$. Using this property find L[sinh(at) sin(at)].

(3) Find Laplace transform of f(t) Where

f(t) = t, where $0 < t \leq 4$

= 5 where t > 4

Que-2(B) Answer any one out of two. **(04)**

(1) Find : $L^{-1}\left[\frac{3(s^2-1)^2}{2s^5}\right]$.

(2) Find : $L^{-1}\left[\frac{s+1}{s^2+s+1}\right]$.

Que-3(A) **Answer any two out of three.** **(10)**

(1) Using convolution theorem find $L^{-1}\left[\dfrac{1}{(s^2+a^2)^2}\right]$.

(2) Prove that if $L\{f(t)\}=\bar{f}(s)$ then $L\left[\int_0^t f(u)du\right]=\dfrac{1}{s}\bar{f}(s)$. Using this result find $L^{-1}\left[\dfrac{1}{s^3(s^2+a^2)}\right]$.

(3) If $L\{f(t)\}=\bar{f}(s)$ and $\dfrac{f(t)}{t}$ has Laplace transform then prove that $L\left\{\dfrac{f(t)}{t}\right\}=\int_s^\infty \bar{f}(s)ds$. Using this result find $L\left[\dfrac{1-\cos(2t)}{t}\right]$.

Que-3(B) **Answer any one out of two.** **(04)**

(1) By using integration of Laplace transform find $L\left[\dfrac{e^{-at}-e^{-bt}}{t}\right]$.

(2) Find : $L\left[e^{-4t}\int_0^t \dfrac{\sin 3t}{t}dt\right]$.

Que-4(A) **Answer any two out of three.** **(10)**

(1) Prove that if $K' \leq G'$ then $\varphi^{-1}(K') \leq G$ where $(G,*)$ and $(G',*')$ be two groups and $\phi: G \to G'$ is Homomorphism.

(2) Show that every field is an integral domain. Is the converse true? Explain.

(3) State and prove Factor theorem and Remainder theorem of polynomials.

Que-4(B) **Answer any one out of two.** **(04)**

(1) Prove that factorization of x^4+3x^3+2x+4 in $Z_5[x]$ is $(x-1)^3(x+1)$.

(2) Consider two groups $(\mathbb{C}^*,\cdot)$ and $(\mathbb{R},+)$ where $\mathbb{C}^* = \mathbb{C} - \{0\}$. Define $f: \mathbb{R} \to \mathbb{C}^*$ as $f(x) = e^{ix}$. Show that f is homomorphism and find K_f.

Que-5(A) **Answer any two out of three.** **(10)**

(1) State and prove division algorithm theorem of polynomials.

(2) For non-zero polynomial $f(x), g(x) \in D[x]$, prove that deg(fg)=deg(f)+deg(g).

(3) Define irreducible polynomial. Find irreducible factors of $4x^2-4x+8$ over $\mathbb{Q}$.

Que-5(B) **Answer any one out of two.** **(04)**

(1) Find G.C.D. d(x) of $f(x), g(x) \in \mathbb{Z}_5[x]$ where $f(x)=x^3+2x^2+3x+2$ and $g(x)=x^2+4$. Also express d(x) as a(x)f(x)+b(x)g(x).

(2) If $a=a_1+a_2i+a_3j+a_4k$ is non-zero quaternion number then find a^{-1}. Also check whether set of quaternions is a field or not.

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